1902MNRAS..62..180B

The following Candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

John Waller Lawrence Child, Vernham, Merton Hall Road, Wimbledon (proposed by Col. E. E. Markwick) ;
José Comas Solá, S. Felipe, 29 S. Gervasio, Barcelona, Spain (proposed by A. Stanley Williams) ;
Rev. John Horsley Haslam, M.A. Cantab., Vicar of St. Saviour's, Denmark Park, S.E. (proposed by D. G. Simpson) ;
Joseph Lunt, B.Sc., F.I.C., Royal Observatory, Cape of Good Hope (proposed by Sir David Gill) ; and
Joges Chandra Ray, M.A., Professor of Physical Science, Cuttack, Bengal, India (proposed by U. Chandra Ghosh).

Seventy-six presents were announced as having been received since the last meeting, including amongst others :—

Galileo, Opere, Edizio Nazionale, vol. 11, presented by the Italian Government ; 22 Charts of the international photographic chart of the heavens, presented by the Paris Observatory ; Wedgwood Medallion of Sir Wm. Herschel by Flaxman, presented by Sir W. J. Herschel ; Photograph from a portrait of Galileo by Sustermans, presented by Sir W. Huggins.

The Attraction of the Himalaya Mountains upon the Plumb-line in India.[*] By Major S. G. Burrard, R.E., Superintendent of the Trigonometrical Survey of India.

1. In a paper read in 1895 before the Royal Society[†] the late General Walker advocated the employment of "groups" of astronomical stations for the purpose of eliminating the effects of local attraction at a central station. In the same paper he explained the preponderance of northerly deflections that prevails throughout India by the hypothesis that local attraction is producing a southerly deflection at Kalianpur, *the station of reference of the Indian Survey.*[‡] In 1871 Archdeacon Pratt had come to the same conclusion ; in his "Figure of the Earth" he argues that an excess of density must exist near Kalianpur, for "the deflection

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[*] For further details see Professional Papers of the Survey of India Department.

[†] *Phil. Trans.* clxxxvi. 745.

[‡] No absolute deflection is known for any place ; all deflections are relative to that at Kalianpur. The term "northerly deflection" denotes a deflection "more northerly than the deflection at Kalianpur."

1902MNRAS..62..180B

at Kalianpur," he writes, "is considerably south." Shortly after the issue of General Walker's paper in 1896, Sir David Gill's work on the Geodetic Survey of South Africa was published, in which the adoption of "groups" of astronomical stations was strongly recommended. In consequence of the opinions expressed by these eminent writers, and in full accord with their views, we decided in 1898 to determine the actual effect of local attraction at Kalianpur by means of a complete "group."

2. By the spring of 1899 astronomical observations for both latitude and azimuth had been taken at Kalianpur, and at eight adjacent stations, four being situated at an average distance of nine miles and four at an average distance of thirty-five miles from Kalianpur. A discussion of the results of the observations showed that local attraction was causing a *northerly* deflection at Kalianpur. This unexpected issue created a dilemma; either General Walker had been mistaken in advocating "groups," or his assumption of a southerly deflection at Kalianpur had been incorrect.

3. A classification of the results of the latitude observations discloses two remarkable facts: First, that two-thirds of the southerly deflections observed in India are situated within a narrow zone, running from east to west between the parallels of 24° and 26° approximately. Second, that large northerly deflections prevail from east to west between the parallels of 24° and 18° approximately.

4. I attached at first great significance to the fact that the parallel of 24°, along which approximately the direction of the deflection changes sign, happens to be the parallel of Kalianpur, the station of reference of the survey. I calculated the respective effects of the Himalayan mass and of the deficiency of matter in the Indian Ocean at twelve different stations, and found that if the Himalayas exercised their normal attraction on the plumb-line throughout India, and the deficiency of matter in the Indian Ocean exerted a northerly repulsive force, the sign of the deflections would change along the parallel of 24°. But this explanation of the observed phenomena could not be maintained; the results of the longitude arcs observed across India from coast to coast showed that the deficiency of matter in the Indian Ocean was wholly compensated by submarine strata of excessive density. If the effect of the Indian Ocean is compensated in the prime vertical it must be compensated also in the meridian.

5. After this first failure I undertook an investigation to see whether the change in the sign of the deflection could be due to errors of the ellipsoid of reference; but, though such errors would produce a change of sign along the parallel of the station of origin, the investigation showed that no considerable modification of the Clarke spheroid was possible.

6. The difficulty with which we are now confronted is this: it is clear that Himalayan attraction is *not* compensated so completely as Archdeacon Pratt believed, large deflections of

1902MNRAS..62..180B

35″, 37″, 38″ having been met with in sub-Himalayan regions since the Archdeacon's death; but if Himalayan attraction is affecting the plumb-lines in India, the deflections north of latitude 24° should be more northerly than the deflection at Kalianpur, and the deflections south of latitude 24° should be less northerly. As has been explained above the observed deflections north of Kalianpur are southerly, and the observed deflections south of Kalianpur are northerly. (The deflections cannot be regarded as accidental or as due to "local attraction.")

7. To illustrate the observed phenomena the following table is given in which typical stations only have been entered. The meridian of 77° 30′ has been selected, but the same alternation of signs is observed on other meridians.

Table I.

Station.		Lat.		Observed deflection.	
		°	′	″	
Dehra Dun	...	30	19	−38	Himalayan influences.
Kaliana ...	...	29	31	− 7	
Noh ...	...	27	51	0	Belt of southerly deflections.
Daiadhari	...	24	38	+ 1	
Kalianpur	...	24	7	station of reference.	
Ladi ...	...	23	8	− 5	Belt of large northerly deflections.
Badgaon ...	...	20	44	− 8	
Damargida	...	18	3	− 3	
Cape Comorin	...	8	9	+ 2	

Under the Himalayas at Dehra Dun we find deflections of 38″; within sixty miles at Kaliana they dwindle to 7″; and in latitude 27° the deflections are southerly; south of Kalianpur at Ladi and Badgaon we meet with a succession of northerly deflections, which gradually become smaller as we proceed southwards to Cape Comorin.

8. The large deflections at Dehra Dun have hitherto not received the attention they deserve; they have been brushed aside with the remark, "At the foot of the Himalayas."

If Himalayan attraction is capable of producing a deflection of 38″ at Dehra Dun, its effects must be felt at Cape Comorin; on the other hand, if Himalayan attraction exercises no effect on plumb-lines south of Kaliana, it cannot produce a deflection of 38″ at Dehra Dun.

If a hemisphere of lead existed immediately north of Dehra Dun it might produce a large deflection there, and yet exercise no appreciable effect at Kaliana. But in the case of mountains—and the Himalayas are no exception—the vertical dimension is always small, and the change in elevation always gradual, and no overwhelming excess of matter can exist in the immediate

1902MNRAS..62..180B

vicinity of a station situated even at their foot. In the *Philosophical Transactions of the Royal Society*, vol. 186, p. 803, General Walker wrote:—"It is thus evident that the effect of the attraction of mountain masses on the plumb-line, which may be very large in the immediate vicinity of the mountains, will be reduced at a distance in greater proportion than is assigned by an incomplete application of the law of gravitation, because of the deficiency in the density of the strata under the mountains, which has not hitherto been allowed for. Eventually a point must be reached at which the positive attraction of the matter above will be cancelled by the negative attraction of the deficiency below, and then the mountain masses will have no influence on the plumb-line."

When, however, one comes to apply in practice the theory propounded in this extract, one finds that if the Himalayas are compensated by underlying deficiencies of matter, the compensating cause will affect the plumb-lines at *both near and distant* stations.

9. I have calculated the effect of the attraction of the Himalayan mass at five stations on the meridian of 77° 30′, viz. Dehra Dun, Kaliana, Kalianpur, Damargida, and Cape Comorin, and have interpolated its effects at other stations from a plotted curve. The results are as follows :—

Table II.

Station.	Lat.	Calculated deflection due to— Himalayan ranges.	Tibetan Plateau.	Total.	Differential deflection from Kalianpur.
	° ′	″	″	″	″
Dehra Dun	... 30 19	19	53	72	−54
Kaliana ...	... 29 31	2	34	36	−18
Noh ...	... 27 51	1	27	28	−10
Daiadhari ...	... 24 38		20	20	− 2
Kalianpur ...	... 24 7		18	18	...
Ladi ...	... 23 8		16	16	+ 2
Badgaon ...	... 20 44		13	13	+ 5
Damargida	... 18 3		10	10	+ 8
Cape Comorin	... 8 9		3	3	+15

10. In the following table the results of observation and calculation are compared :—

Table III.

Station.	Differential Deflections. By Observation. =O.	By Calculation. =C.	Discrepancy. =C−O.
Dehra Dun ...	−38	−54	−16
Kaliana ...	− 7	−18	−11
Noh	0	−10	−10

1902MNRAS..62..180B

Station.	Differential Deflections. By Observation. =O.	 By Calculation. =C.	Discrepancy. =C—O.
Daiadhari ...	+ 1	− 2	− 3
Kalianpur ...	...	...	...
Ladi	− 5	+ 2	+ 7
Badgaon ...	− 8	+ 5	+13
Damargida ...	− 3	+ 8	+11
Cape Comorin ...	+ 2	+15	+13

11. At stations immediately north of Kalianpur the deflection by theory is northerly, but the observed deflection southerly; at stations immediately south the observed deflection is northerly and the theoretical deflection southerly. There appears therefore to be some hidden cause which is tending to reduce the northerly deflections at northern stations and to increase the northerly deflections at southern stations; and I have been led to infer the existence of an underground chain of excessive density that crosses Central India for 1,000 miles, and which underlies Balasore, Mandla, Bhopal, and Jodhpore. The attraction of this chain at stations situated to the north of it is opposed to Himalayan attraction; at stations situated to the south of it the two attractions act in concert. The chain imposes a wave on the curve of Himalayan attraction, and masks the true effects of the latter. If the hypothesis of the chain is correct, then the change in the sign of the deflections that occurs approximately along the parallel of 24° is not due in any way to the fact that this is the parallel of the station of reference. The proximity of the station of reference to the line of the chain appears, indeed, to be accidental. The significance that I attached to it in the early stages of this investigation appears to have been undeserved.

12. If a line exists towards which all plumb-lines from both sides throughout its entire length are deflected, the conclusion that it overlies a source of attraction may seem obvious, but it is not obvious. The hidden chain of Central India is so large that its presence has escaped detection. The idea that deflections of the plumb-line are accidental and due to "*local* attractions" is general, and every unexplained deflection is involuntarily attributed to a cause hidden *in the immediate* vicinity. When on flat plains we meet with a southerly deflection in latitude 27° and a northerly deflection in latitude 18°, we habitually prefer to assume two separate *local* hidden causes, rather than ascribe both deflections to a distant central cause.

13. The position of the underground chain coincides in places with the Vindhyan Mountains; but these latter are insufficient to produce the effects attributed to the chain; moreover, the effects of the chain continue to be observed to the east and to the west after the Vindhyas have ceased to be visible.

14. One feature of the chain is worthy of note: its course

1902MNRAS..62..180B

can be traced from longitude 88° to longitude 73°, and runs from E.S.E. to W.N.W., *in apparent parallelism with the Himalayas.* In the Punjab, to the *west* of the Himalaya Mountains, the longitude arc, Amritsar—Mooltan, reproduces the same extraordinary result as the latitude observations have given in Bengal to the *south* of the Himalayas. Just as the plumb-lines on all meridians from 88° to 73° seem to be deflected inwards towards one line, so are the plumb-lines at Amritsar and Mooltan deflected inwards towards the low-lying alluvium and away from the mountain masses. A chart is attached to illustrate the course of the chain (Plate 6).

15. It cannot be assumed that the discrepancies shown in Table III. are the actual effects of the chain. It is possible that the Himalayan mass is compensated to some extent by subjacent deficiencies of matter. The numerical discrepancies in Table III. have been derived without any such compensation being assumed.

16. In the following table are shown the discrepancies that occur between theory and observation if we assume the attraction of the Himalayas to be compensated to the extent of one-third, one-half, and two-thirds its normal force respectively. As Kalianpur is under the influence of the supposed chain, Cape Comorin has been now taken as the station of reference.

TABLE IV.

Station.	Discrepancies between theory and observation, the attraction of the Himalayan ranges and Tibetan plateau being assumed compensated to the extent of		
	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$
Dehra Dun	− 6	+ 6	+17
Kaliana	−13	− 7	− 2
Noh	−15	−10	− 6
Daiadhari	−10	− 7	− 5
Kalianpur	− 8	− 5	− 3
Ladi	− 2	+ 1	+ 3
Badgaon	+ 3	+ 5	+ 7
Damargida	0	+ 2	+ 3
Cape Comorin	—	—	—

It will be seen that discrepancies of 13″ and 15″ continue to appear, if a theory of a one-third compensation be adopted. The theory of a two-thirds compensation renders the effects of the chain approximately equal and opposite, but introduces a discrepancy of 17″ at Dehra Dun.

17. In the following table are shown the discrepancies that occur between theory and observation if we assume the attraction of the successive Himalayan ranges to be uncompensated and that of the Tibetan plateau to be compensated to the extent of one-third, one-half, and two-thirds its normal force respectively :—

1902MNRAS..62..180B

Table V.

Station.	Discrepancies between theory and observation, the attraction of the Tibetan plateau being assumed compensated to the extent of		
	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$
Dehra Dun	−12	− 4	+4
Kaliana	−14	− 9	−4
Noh	−15	−11	−8
Daiadhari	−10	− 8	−5
Kalianpur	− 8	− 6	−3
Ladi	− 2	+ 1	+3
Badgaon...	+ 4	+ 5	+7
Damargida	+ 1	+ 2	+3
Cape Comorin	—	—	—

The hypothesis that the attraction of the outer Himalayan ranges are uncompensated, and that that of the Tibetan plateau is compensated to the extent of two-thirds its normal force, removes gross discrepancies and renders the effects of the chain at equidistant stations to the north and south of it equal and opposite. It makes the crest of the chain cut the meridian of 77° 30′ in latitude 23° 30′, and it attributes to the chain a maximum effect of 8″ at 200 miles north and south of its crest.*

18. This attempt to disentangle the effects of the chain from those of the Himalayas, and to obtain a definite numerical idea, is put forward more as an illustration of the data than as an exact solution of the problem. The hypothesis that the effects of Himalayan attraction are probably entangled with those of a chain running parallel to the Himalayas at an average distance of 400 miles affords, however, an explanation of the large deflections at Dehra Dun, of the southerly deflections north of latitude 24°, and of the northerly deflections south of latitude 24°.

On Periodic Orbits. By E. T. Whittaker.

The object of this note is to establish the results numbered (I) and (II) below: they relate to the theory of periodic solutions of the differential equations of dynamics, a theory which the researches of Hill, Poincaré, and Darwin have shown to be of great importance in dynamical astronomy.

Consider the motion of a particle in a plane, under the action of conservative forces. Let x, y be its coordinates at time t, referred to any fixed rectangular axes; its mass may be taken

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* The discrepancies at the seven central stations are assumed to be due to the chain.